Correct usage of quantities, units and equations (II)

Quantities

Physical phenomena are described qualitatively and quantitatively by physical quantities. Every value of a quantity can be represented as the product of numerical value and unit. Any variation of the unit (eg prefix added to unit) yields a different numerical value. The product of numerical value and unit remains constant as it is invariant towards a variation of the unit. Example: $V = 0.1$ V and $V = 100$ mV denote the same value of quantity.

Letter symbols for physical quantities are stipulated by DIN 1304 and other standards. Letter symbols are to consist of one letter only, since the use of groups of letters would lead to their misinterpretation in equations as the product of several quantities. For the same reason, abbreviations of names made up of several letters are not to be used to denote a quantity either. To signify the special meaning of a letter symbol, an index in the form of letters or numerals may be added to it.

Quantities of the same kind are given in the same unit. They are distinguished by using different letter symbols or the same letter symbol with index. TABLE 5 shows some examples of quantities of the same kind.

Quantity		SI unit	
Name	Letter symbol	Name	Symbol
Length	l	Meter	m
Width	b	Meter	m
Height	h	Meter	m
Frequency	f	Hertz	Hz
Resonance frequency	f_r, f_{rsn}	Hertz	Hz
Bandwidth	B, f_B	Hertz	Hz
Electric voltage	V	Volt	V
RMS value of voltage	V_{rms}	Volt	V
Power	P	Watt	W
Signal power	P_s	Watt	W
Noise power	P_n	Watt	W
Active power	P, P_p	Watt	W
Reactive power	Q, P_q	Watt	W (also Var)
Apparent power	S, P_s	Watt	W (also VA)

TABLE 5 Examples of quantities of same kind

Equations

The German expressions for quantity equation (Größengleichung), scaled quantity equation (zugeschnittene Größengleichung), numerical-value equation (Zahlenwertgleichung) as well as the relation value_of_quantity = numerical_value_·_unit are based on the work of Julius Wallot between 1922 and 1933. Discussions on this topic culminated in the first edition of standard DIN 1313 (1931): Notation of physical equations.

Quantity equations (DIN 1313) are equations where the letter symbols represent physical quantities or mathematical symbols (numerals, variables, functions, operators). Quantity equations are independent of the units chosen. When evaluating quantity equations the letter symbols are to be replaced by the products of numerical values and units. Numerical values and units in quantity equations are treated as independent factors. Example: the equation

$$V = R \cdot I$$

always yields the same result irrespective of the units used for resistance R and current I, provided the associated products of numerical value and unit are substituted for R and I.

Scaled quantity equations (DIN 1313) are quantity equations where every quantity appears with its unit in the denominator. Example:

$$V/\text{kV} = 10^{-3} \cdot (R/\Omega) \cdot (I/\text{A})$$

The parentheses can be omitted if the assignment of quantities and units is unambiguous, as for example on the left of the equation above or when horizontal bars are used.

$$\frac{V}{\text{kV}} = 10^{-3} \, \frac{R}{\Omega} \, \frac{I}{\text{A}}$$

The scaled quantity equation has the advantage that the quotients of quantity and unit represent the numerical values related to the given units. The equations remain correct if the products of numerical values and units in some other units are substituted for the quantities. In this case however, the units must be converted. The scaled quantity equation is mainly used for representing results.

Numerical-value equations should not be used any more since they have been considered outdated for over 60 years. According to DIN 1313 they need to be specially marked as numerical-value equations and units must be specified for all quantities.

According to the standards, units in brackets are not to be added to the quantity symbols within equations. Unfortunately, bracketing is widely spread – it can even be found in manuscripts of university and college teachers. Example not to be followed:

$$V\,[\text{kV}] = 10^{-3} \cdot R\,[\Omega] \cdot I\,[\text{A}]$$

☹ *wrong*

DIN 1313 rules out this notation. If the quantities in this equation are expressed as products of numerical values and units, the equation does not make sense, since the units appear twice as factors. The scaled quantity equation should be used whenever the relation between numerical values is to be shown.

Logarithmic quantities, attenuation and gain

Logarithmic quantities relate to **logarithmic ratios** of powers or field quantities that define the characteristics of an object (twoport, eg transmission element) [8]. Decibel (dB) is the unit used.

Definition for field quantities (eg for the complex amplitudes of AC voltages):

Voltage attenuation

$$A_V = 20 \lg \left| \frac{V_1}{V_2} \right| \text{dB}$$

Voltage amplification, voltage gain

$$G_V = 20 \lg \left| \frac{V_2}{V_1} \right| \text{dB}$$

Definition for real power quantities (eg active power):

Power gain

$$G_P = 10 \lg \frac{P_2}{P_1} \text{dB}$$

The arguments of the logarithm are quantities of dimension 1 (numerical values). The unit dB is also of dimension 1 and is therefore designated as a pseudo unit. It is not an SI unit. The function lg designates a logarithm to the base 10, log stands for the general logarithmic function.

To be continued.

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REFERENCES

[8] DIN 5493-2: Logarithmische Größen und Einheiten, Logarithmierte Größenverhältnisse, Maße, Pegel in Neper und Dezibel (09/94)

Correction: Table 3 of part I of this refresher topic erroneously gives Θ as the symbol for magnetic flux. The correct symbol is Φ.